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Seat No. ________________

HAN-003-1163005

M. Sc. (Sem. III) Examination

June – 2023

Mathematics : EMT-3011

(Differential Geometry)

Faculty Code : 003
Subject Code : 1163005

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions :

(1) There are total five questions.

(2) Each question carries equal marks.

(3) All the questions are compulsory.

1 Attempt any **seven** : **14**

(1) Define with example : Functions of class k.

(2) Define with example : Simple surface.

(3) Find the radius and curvature of the circle

$$3x^2+3y^2-24x-12y+12=0\cdot$$

(4) Is the curve $\alpha(x)=(x^{99},2x+5,5x^2+7)$ is regular ? Justify your answer.

(5) Define : Unit speed curve.

(6) Define : The tangent plane and the normal plane.

(7) Define : Normal curvature and Geodesic curvature. Also state the relation between k, k_n and k_g.

(8) Write the $p-\alpha$ form of the equation of line and find its curvature.

(9) Is the surface $(x, y, 2x+3y)$ simple ? Justify your answer.

(10) Define : Right circular helix.

2 Attempt the following : **14**

(a) If $g:[c,d]\to[a,b]$ is a reparametrization of a curve segment $\alpha:[a,b]\to R^3$ then prove that the length of α is equal to the length of $\beta=\alpha\circ g$. Also derive the relation between their tangent planes.

(b) Is the curve $\alpha(t)=(\sin 6t\cos t,\sin 6t\sin t,0)$ regular ? If so then find the equation of tangent line to α at $t=\frac{\pi}{6}$.

OR

(a) Find the length of the curve

$\alpha(t)=\left(2a\left(\sin^{-1}t+t\sqrt{1-t^2}\right),2at^2,4at\right)$ between the point $t=5$ to $t=10$.

(b) Define the arc length of a curve and prove that the arc length is one-one function mapping (a,b) onto (c,d) and it is a reparametrization.

3 Attempt the following : **14**

(a) Let $\alpha(s)$ be a unit speed curve whose image lies on a sphere of radius r and centre m then show that $k\neq 0$. Also if $\tau\neq 0$ then $\alpha-m=-\rho N-\rho'\sigma\beta$ and $r^2=\rho^2+(\rho'\sigma)^2$ (where $\rho=\frac{1}{k}$ and $\sigma=\frac{1}{\tau}$).

(b) Is the curve $\alpha(t)=(\sin t,\cos^2 t,\cos t)$ regular ? If so then find the equation of tangent line at $t=\frac{\pi}{4}$.

OR

(b) Find the arc length of the curve $\alpha(t)=(r\cos t,r\sin t,0)$ and reparametrize the curve by its arc length. For the circular helix $\alpha(t)=(r\cos\omega s,r\sin\omega s,h\omega s)$, compute Frenet – Serret apparatus (where $\omega=\left(r^2+h^2\right)^{\frac{1}{2}}$).

4 Attempt the following : **14**

(a) Show that the curve $\alpha(S)=\left(\frac{5}{13}\cos S, \frac{8}{13}-\sin S, -\frac{12}{13}\cos S\right)$ is a unit speed curve. Also compute its curvature and torsion.

(b) Let $f: X \to R^3$ be a simple surface and $f: v \to u$ is a co-ordinate transformation then prove that $y = X \circ f : v \to R^3$ is also a simple surface.

5 Attempt any **two** : **14**

(a) List the coefficients of first fundamental forms and find the value of determinant of (g_{ij}) for the metric

$$ds^2 = dr^2 + r^2 \sin^2 \theta d\theta^2 dr^2 + d\theta^2$$

(b) For a simple surface $x: u \to R^3$ prove that

(i) $x_{ij} = L_{ij} n + \sum_k \Gamma_{ij}^k x_k$

(ii) For any unit speed curve

$$\gamma(S) = x(\gamma'(S), \gamma^2(S)), k_n = \sum_{i,j} L_{ij} (\gamma^i)'(\gamma^j)' \text{ and}$$

$$k_g S = \sum_k \left[\left(\gamma^k\right)'' + \sum_{i,j} \Gamma_{ij}^k (\gamma^i)' (\gamma^j)' \right] x_k.$$

(c) For the surface $x(u^1, u^2) = (u^1, u^2, f(u^1, u^2))$ compute coefficients of second fundamental form and Christoffel symbols for the same.

(d) State and prove Frenet – Serret theorem.
